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COMBINATORIAL ASPECTS OF MESSAGE FLOW IN PACKET RADIO NETS

PART III

Contents

I. Introduction

II. Outline of Problems

III. Useful Results Previously Obtained

IV. Analysis of Problem I

V. Analysis of Problem II

VI. Outline of Computer Analysis

VII. Summary, Conclusions and Future Problem Areas

I. INTRODUCTION

This report is the third in the series "Combinatorial Aspects of Message Flow in Packet Radio Nets". In the second of the series we carried out a combination theoretical-computer study of message flow into a ground station from both an infinite and finite net of repeaters. The repeaters originate messages and repeat all messages which are "accepted".

One problem area which we considered theoretically in the previous analyses was to compute the probability distributions of the number of copies of a message which are accepted at a ground station (origin in coordinate net) of a single message which originated d units away from the station. Some computer generated data is given on that problem based on a theoretical model which we describe in Sections V and VI of this report.

A next stage in the analysis of a packet radio net is to consider traffic being generated from the ground station at a constant rate and repeated outward to all repeaters. We discuss the analysis in the infinite grid of several aspects of this problem. Our basic assumptions for the outward part of the model are:

A. We again take as our physical placement of repeaters and ground station the Euclidean plane. A repeater is located at each lattice point of the grid.

B. Starting at time t = 0 and at quantized time periods afterwards, (perhaps 1 second) denoted by t = 1, 2, ...; τ messages originate at the fixed ground station. The ground station is physically considered to be at the origin. The τ messages are distributed into the m-slots (as earlier in Part II of the series), and are transmitted according to the mode 1 and mode 2 capture criteria.

C. At each repeater, at each time point some number of messages arrive and some number are accepted. The relationship between the numbers which arrive and are accepted is determined by the random mechanism of the slotting-capture system. Of the number of messages which are accepted some fixed proportion, k_0, is assumed to be addressed to that repeater and are not repeated. The remaining accepted messages are repeated to each repeater at distance one unit further from the given repeater, and are received one time unit later.

II. OUTLINE OF PROBLEMS

We will discuss and offer solutions to two specific problems. The first problem which we solve is concerned with analysis of messages being repeated towards the origin. Hence, problem number one is a continuation of the study of the packet radio net with messages only being sent to the ground station.

PROBLEM AREA I:

Develop a theoretical model which studies the dynamics of a single message which originates at a given repeater d units from the station in terms of the number of copies of the message which arrive at repeaters along the way. In particular, determine the probability distribution of the number of copies which arrive and are accepted at the origin. Adapt this model to a computer analysis so that specific probabilities can be determined as a function of distance of originator from the ground station. Develop numerical data.

In the second problem area we begin development of a new aspect of message flow, namely flow outward from the ground station to the various repeaters. In this report we study the infinite boundary case and develop several formulae for message flow.

PROBLEM AREA II:

Under the conditions of message flow from (outward) the ground stations to the repeaters, compute the distribution of the number of messages arriving and accepted at each repeater.

III. USEFUL RESULTS FROM PREVIOUS ANALYSES

In "Combinatorial Aspects of Message Flow in Packet Radio Nets" Part II, we obtained useful theoretical formulae which relate message arrivals to message acceptances at repeaters. Two modes were considered from the point of view of capture design. The two modes, which we denote by mode 1 and mode 2, are respectively:

1. If two messages arrive in the same "slot" neither is accepted.
2. The number of messages accepted is given by the number of non-empty slots.

We derived the following formulae, where P_{kj} and P^*_{kj} denote respectively the probability that j messages are accepted given that k arrive in mode 1 and mode 2. We recall:

$$(1) \qquad P_{kj} = \frac{\binom{m}{j}}{m^k} \sum_{\nu=0}^{\min(k,m)-j} (-1)^{\nu} \binom{m-j}{\nu} \frac{k!}{(k-j-\nu)!} (m-j-\nu)^{k-j-\nu}$$

when $j \leq \min(k,m)$ and zero otherwise.

When $k \geq 20$ "approximately" (1) can be approximated by:

$$(2) \qquad P_{kj} = \frac{e^{-\lambda_1} \lambda_1^j}{j!} \quad \text{where} \quad \lambda_1 = ke^{-\frac{k}{m}}$$

for $j \leq \min(k,m)$ and zero otherwise.

For P^*_{kj} we have the corresponding formulae for mode 2:

$$(3) \qquad P^*_{kj} = \sum_{\nu=0}^{j} \binom{m}{j} (-1)^{\nu} \binom{j}{\nu} \left(\frac{j-\nu}{m}\right)^k$$

when $0 \leq j \leq \min(k,m)$, and zero otherwise.

When $k \geq 20$ (3) can be approximated by:

$$(4) \qquad P^*_{kj} = \frac{e^{-\lambda_1} \lambda_1^j}{j!} \quad \text{where} \quad \lambda_1 = ke^{-\frac{k}{m}}$$

when $0 \leq j \leq \min(k,m)$, and zero otherwise.

IV. ANALYSIS OF PROBLEM I

In this section we will develop the theoretical basis for a computer analysis of the dynamics of a single message originated in the net, and attempting to reach the ground station at the origin.

We carry out a more detailed analysis related to the results of section 5 of "Combinatorial Aspects of Message Flow in Packet Radio Nets" Part II, which is

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directed towards a computer analysis. Let us assume that the given message originates at a repeater with coordinates (i,j) at time t. If the incoming and acceptance numbers at (k,j) at time t are respectively $X_{(ij)}(t)$ and $X^{A}_{(ij)}(t)$ we assume the given message is one of the $X_{(ij)}(t)$ messages. Furthermore we assume that each of the $X_{(ij)}(t)$ messages is equally likely to be one of the accepted messages. Under these assumptions it follows that at (i,j), there are two types of messages which have arrived. The first type is one message (the given one), the second type are $X_{(ij)}(t)-1$ messages. The probability of acceptance at (i,j) is given by the hypergeometric probability density function:

$$(1) \qquad \frac{\binom{X_{(ij)}(t)-1}{X^{A}_{(ij)}(t)-1}}{\binom{X_{(ij)}(t)}{X^{A}_{(ij)}(t)}}$$

At each repeater on every path to the ground station the same analysis applies. At any given repeater, on the path, say with coordinates (k,e) there may be several copies of the original message which arrive.

Suppose (k,e) is on a path from (i,j) to (0,0) and the number of paths from (i,j) to (k,e) is w. Then at (k,e), at time t plus the distance from (i,j) to (k,e), either 0, 1, 2, ..., up to w copies of the message may arrive. If d is the distance from (i,j) to (k,e) and at time t + d, $X_{(k,e)}(t+d)$ and $X^{A}_{(k,e)}(t+d)$ messages respectively arrive and are accepted then we can compute the probability that exactly Z copies of the original messages are accepted. The computation of the required probabilities is a direct extension of formula one and is given by:

(2) P {exactly Z copies of original message is accepted at (k,e) at time t + d/ν copies are amongst the arrivals}

$$= \frac{\binom{\nu}{Z}\binom{X_{(k,e)}(t+d)-\nu}{X^{A}_{(k,e)}(t+d)-Z}}{\binom{X_{(k,e)}(t+d)}{X^{A}_{(k,e)}(t+d)}} \quad ; \quad Z = 0, 1, 2, \ldots, \nu.$$

Equation (2) is valid at every repeater along every path from (i,j) to (0,0), and in particular at the origin. The only ingredient needed to apply the equations to a computer analysis and generate numerical values is a formula for the probability that exactly v copies of the message arrive at each repeater. This formula can be obtained recursively using the idea of isodesic line and wedge joint density functions and developed in "Combinatorial Aspects of Message Flow in Packet Radio Nets" Part II in section six.

If a single copy of the given message is accepted at its origination repeater, it is:

a) repeated to each of two repeaters one unit closer to the origin if it is not on an axis;

b) repeated to the one repeater one unit closer to the origin if it is on an axis.

We will focus only on a) since b) is essentially identical as far as the analysis is concerned. The message, when accepted at its origination, is then repeated to repeaters at (i-1,j-1) and (i-1,j). Acceptances at (i-1,j-1) and (i-1,j) are determined according to Equation (2). The isodesic line joint density of receptions and acceptances are computed at (i-1,j-1) and (i-1,j). This joint density then determines arrivals and acceptances at (i-2,j-2), (i-2,j-1) and (i-2,j). The process then continues recursively until all computations are carried out at the origin.

A sample of these computations is carried out for messages originating at repeaters which are 5,4,3,2,1,0 units of distance from the origin in section VI of this report. Numerical data is obtained for the distribution of the number of copies of a given message received at the origin as a function of distance. The expected number of copies is also computed and graphed. Comparison tables for two modes (1 and 2) of capture are developed. Significant differences arise as would be expected. In addition, comparisons are made as a function of λ the mean number of messages which originate at each repeater at each point in time. The values of λ which are studied are 1,3,5 for slots of size 100.

V. ANALYSIS OF PROBLEM II - BASIC EQUATIONS

The next large part of our analysis of the combinatorial aspects of message flow is to study the flow of messages being repeated outward from the origin.

We assume that at each point of time t, τ messages are being generated outward from the origin. Of messages accepted at each repeater, a fixed proportion k_0 are addressed to that repeater and hence are not repeated. We study the distributions of the number of messages received and accepted at each repeater at each point in time, assuming an infinite net.

Recall $X^{(1)}_{(ij)}(t)$ = number of messages arriving at a repeater with coordinates (i,j) at time t with mode 1 capture.

$X^{A,(1)}_{(ij)}(t)$ = number of messages accepted at a repeater with coordinates (i,j) at time t in mode 1 capture. In mode 2 capture, we use the same notation except that (1) as a superscript is replaced by a (2).

A. IN MODE ONE:

In mode one capture, the relationship between arriving and accepted messages is described by the transfer function:

$$(1) \qquad P_{kj} = \frac{\binom{m}{j}}{m^k} \sum_{\nu=0}^{\min(k,m)-j} (-1)^\nu \binom{m-j}{\nu} \frac{k!}{(k-j-\nu)!} (m-j-\nu)^{k-j-\nu}$$

when $j \leq \min(k,m)$, and zero otherwise.

We can study $X^{(1)}_{(ij)}(t)$ and $X^{A,(1)}_{(ij)}(t)$ recursively.

At the origin for each t = 0, 1, 2, ...; $X^{(1)}_{(0,0)}(t) = \tau$, a fixed constant. Furthermore:

$$(2) \qquad P\{X^{(1),A}_{(0,0)}(t) = w\} = P_{\tau,w} = \frac{\binom{m}{w}}{m^\tau} \sum_{\nu=0}^{\min(\tau,m)-w} (-1)^\nu \binom{m-w}{\nu} \frac{\tau!}{(\tau-w-\nu)!} (m-w-\nu)^{\tau-w-\nu}$$

If we assume $\tau \leq m$;

$$(3) \qquad P\{X^{(1),A}_{(0,0)}(t) = w\} = \begin{cases} \dfrac{\binom{m}{w}}{m^\tau} \displaystyle\sum_{\nu=0}^{\tau-w} (-1)^\nu \binom{m-w}{\nu} \frac{\tau!}{(\tau-w-\nu)!} (m-w-\nu)^{\tau-w-\nu} & \text{if } w \leq \tau \\ 0 & \text{if } w > \tau. \end{cases}$$

At coordinates (1,1), (1,0) the distribution of $X_{(1,0)}(t)$ and $X_{(1,1)}(t)$ are identical, hence we write only $X^{(1)}_{(1,0)}(t)$.

We have:

$$(4) \qquad X^{(1)}_{(1,0)}(t) = \begin{cases} X^{(1),A}_{(0,0)}(t-1) & t = 1, 2, \ldots; \\ 0 & \text{if } t = 0. \end{cases}$$

For the acceptance at $t = 1, 2, \ldots;$

$$P\{X^{A,(1)}_{(1,0)}(t) = j\} = \sum_{k=j}^{\tau} P_{kj} \cdot P\{X^{(1)}_{(1,0)}(t) = k\} \qquad j = 0, 1, 2, \ldots;$$

or recursively:

$$(5) \quad P\{X^{A,(1)}_{(1,0)}(t) = j\} = \begin{cases} \sum_{k=j}^{\tau} P_{kj} \cdot P\{X^{(1),A}_{(0,0)}(t-1) = k\} & \text{if } j \leq \tau; \\ 0 & \text{otherwise.} \end{cases}$$

Equation (5) is recursively solvable since $P\{X^{(1),A}_{(0,0)}(t-1) = k\}$ is given by (3) and P_{kj} is given by (1).

Now more generally at a repeater at distance d with coordinates (d,j); $j \neq 0,d$. We have for $d = 2, 3, \ldots;$

$$(6) \qquad X^{(1)}_{(d,j)}(t) = (1-k_0)\left[X^{(1),A}_{(d-1,j-1)}(t-1) + X^{(1),A}_{(d-1,j)}(t-1)\right].$$

The acceptances are given by:

$$(7) \quad P\{X^{A,(1)}_{(d,j)}(t) = r\} = \sum_{k=r}^{(2m)} P_{k,r} \cdot P\{X^{(1)}_{(d,j)}(t) = k\};$$

where P_{kr} is given by (1) and $P\{X^{(1)}_{(d,j)}(t) = k\}$ can be computed recursively from (6) using the notation of isodesic line joint densities as developed in "Combinatorial Aspects of Message Flow in Packet Radio Nets" Part II in section six. The equations for mode 2 analysis are identical except that P_{kj} is replaced by P^{*}_{kj}.

When j = 0 or d, i.e. the repeater is on the axis at distance d a simpler analysis unfolds. Since the random variables $X_{(d,0)}(t)$ and $X_{(d,d)}(t)$ have the

same probability distribution we write equations only for $X_{(d,0)}(t)$; $d \geq 2$ since $X_{(0,0)}(t)$ and $X_{(1,0)}(t)$ have already been determined.

$$(8) \qquad X^{(1)}_{(d,0)}(t) = (1-k_0)\, X^{(1),A}_{(d-1,0)}(t-1); \qquad t = d, d+1, \ldots; \quad d = 2, 3, \ldots .$$

For the acceptances;

$$(9) \qquad P\{X^{(1),A}_{(d,0)}(t) = j\} = \sum_{k=j}^{\infty} P_{kj}\, P\{X^{(1)}_{(d,0)}(t) = k\}, \qquad = 0, 1, 2, \ldots, m$$

The equations (1) through (9) can be used with computer generated data to study message arrivals and acceptances at each repeater. This will be done as a "next project".

VI. OUTLINE OF COMPUTER ANALYSIS

In our computer analysis we used the results of section IV to compute the probability distributions and mean value of the number of copies accepted at the origin of a single message which originates at distances of 5,4,3,2,1,0 units from the ground station. For convenience and realism of the numerical results we selected each originating repeater to have the maximum number of paths to the origin. The coordinate system we used for these calculations is given in Figure 1 below,

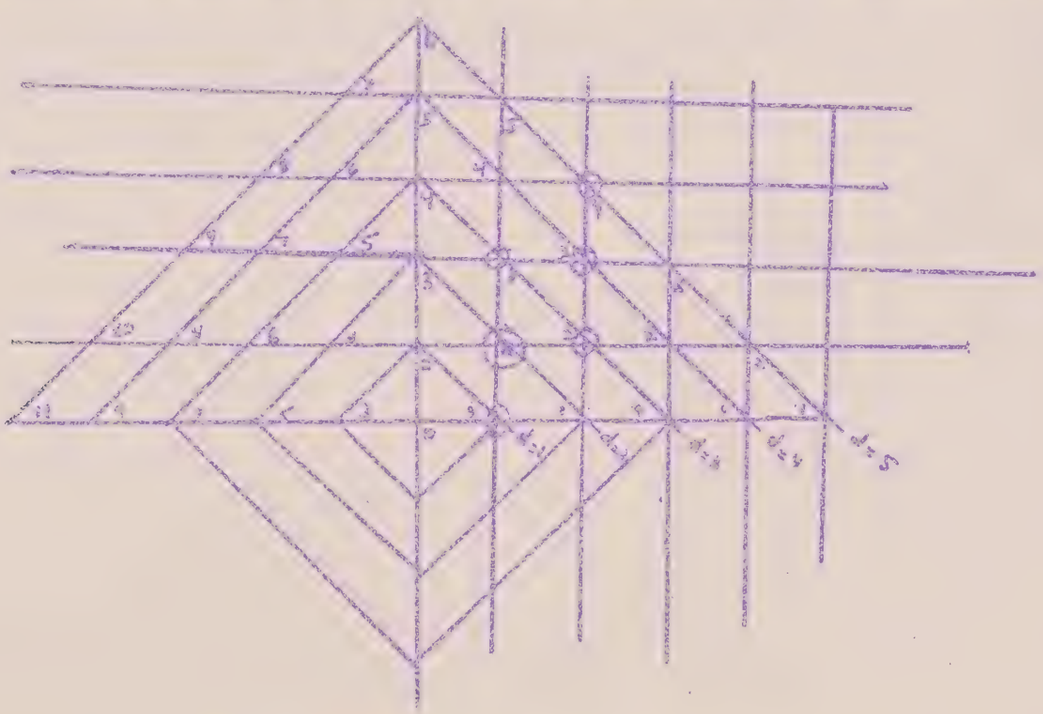

Fig. 1 Coordinate System

The repeaters selected for originating messages at distances 5,4,3,2,1,0 are respectively at (5,4), (4,3), (3,3), (2,2), (1,1), (0,0). The routes are designated in Figure 2, and the maximum number of copies of an originating message which can be received along each repeater on the route is given in Table 1 below. Note that the maximum number of possible copies is given by the number of paths from an originating repeater to the receiving repeater. Note in Table 1 that no copies can be received at a repeater further from the origin than the originator.

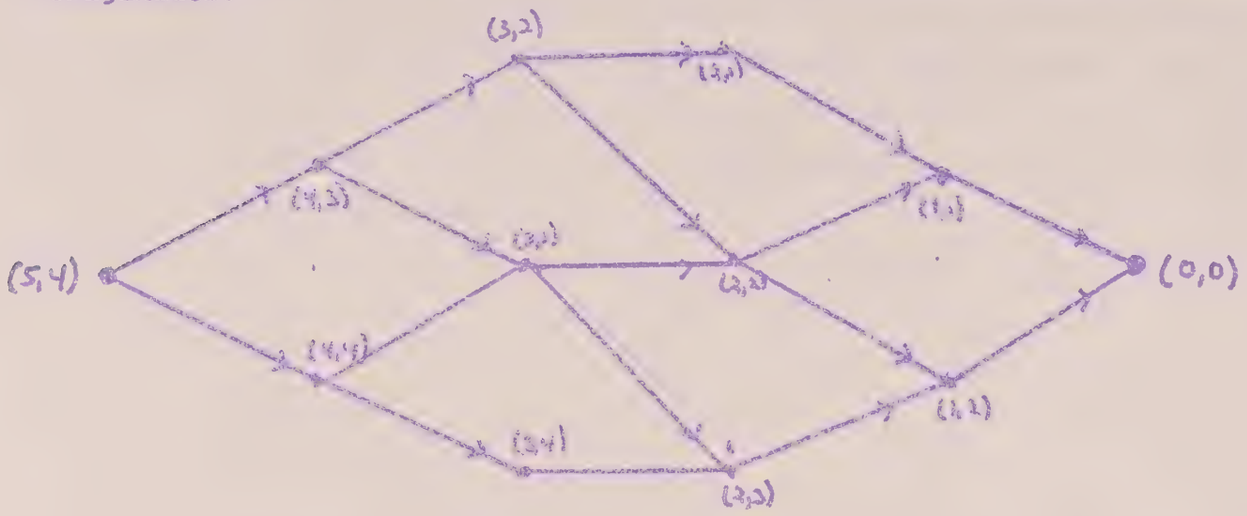

Fig. 2 Routing From (5,4) to (0,0)

	(0,0)	(1,1)	(1,2)	(2,1)	(2,2)	(2,3)	(3,2)	(3,3)	(3,4)	(4,3)	(4,4)	(5,4)
(0,0)	1	1	1	1	2	1	3	3	1	6	4	10
(1,1)		1	0	1	1	0	2	1	0	3	1	4
(1,2)			1	0	1	1	1	2	1	3	3	6
(2,1)				1	0	0	1	0	0	1	0	1
(2,2)					1	0	1	1	0	2	1	3
(2,3)						1	0	1	1	1	2	3
(3,2)							1	0	0	1	0	1
(3,3)								1	0	1	1	2
(3,4)									1	0	1	1
(4,3)										1	0	1
(4,4)											1	1
(5,4)												1

Table 1 Maximum Number of Copies - Between Two Repeaters

The numbers in Table 1 give the upper limits of the summation for the possible copies of messages which can be received at each repeater of a single message originating at a repeater further from the origin but within the net of Figure 2. With the selected net and the numbers of Table 1, we can use the results of section IV to obtain numerical data.

At time zero a random number of messages has arrived at each repeater. To compute the distribution of copies arriving at (0,0) from (5,4) we assume one of the messages arriving at (5,4) is singled out and followed along the route using the hypergeometric analysis of section IV. The procedure was used for $t = 0, 1, 2, \ldots, 40$ in conjuction with the random Poisson number generator developed and discussed in "Combinatorial Aspects of Message Flow in Packet Radio Nets" Part II, in section X.

Specifically we seek to compute the five numbers:

$Z_0(j;t)$; $j = 0, 1$; $t = 0, 1, 2, \ldots, 40$;

$Z_1(j;t)$; $j = 0, 1$; $t = 1, 2, \ldots, 40$;

$Z_2(j;t)$; $j = 0, 1, 2$; $t = 2, 3, \ldots, 40$;

$Z_3(j;t)$; $j = 0, 1, 2, 3$; $t = 3, 4, \ldots, 40$;

$Z_4(j;t)$; $j = 0, 1, 2, 3, 4, 5, 6$; $t = 4, 5, 6, \ldots, 40$;

$Z_5(j;t)$; $j = 0, 1, 2, 3, \ldots, 10$; $t = 5, 6, \ldots, 40$;

where $Z_k(j;t)$ is the probability that exactly j copies of a message originating at a repeater at distance k at time t-k, are accepted at the origin at time t. For the computer analysis we considered one repeater at each of the distances, as in Figure 1. The maximum j values are given by the first row of Table 1. Using the hypergeometric analyses the following equation can be used to compute each of the $Z_k(j;t)$; as a function of:

1) λ = mean number of originations at each repeater.
2) m = number of slots fixed at 100.
3) Each of two capture modes 1 and 2.

For ease of notation we denote:

$$\lambda_{(ij)}(w,x;t) = \binom{\lambda_{(ij)}(t)-w}{\lambda_{(ij)}(t)-x}.$$

A. THE EQUATION FOR $Z_0(j;t)$

Clearly $Z_0(j;t)$ is simply given by

$$(6.1) \qquad Z_0(j;t) = \frac{A_{(0,0)}(1,j;t)}{A_{(0,0)}(0,0;t)} \; ; \; j = 0, 1; \quad t = 0, 1, 2, \ldots, 40.$$

B. THE EQUATION FOR $Z_1(j;t)$

$$(6.2) \qquad Z_1(j;t) = \sum_{\mu=0}^{1} \binom{\mu}{j} \cdot \frac{A_{(0,0)}(\mu,j;t)}{A_{(0,0)}(0,0;t)} \cdot f_1^i(\mu;t-1)$$

for $j = 0, 1; \quad t = 1, 2, \ldots, 40$; where

$$f_1^1(j;t) = \frac{A_{(1,1)}(1,j;t)}{A_{(1,1)}(0,0;t)} \; ; \; j = 0, 1; \quad t = 0, 1, 2, \ldots, 39.$$

C. THE EQUATION FOR $Z_2(j;t)$

$$(6.3) \qquad Z_2(j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^2(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} \; ;$$

for $j = 0, 1, 2; \quad t = 2, 3, \ldots, 40$; where

$$f_2^2(i,j;t) = \sum_{\mu=0}^{1} f_1^2(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(1,1)}(\mu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu,j;t)}{A_{(1,2)}(0,0;t)} \; ;$$

for $t = 1, 2, \ldots, 39; \quad i = 0, 1; \quad j = 0, 1$; where

$$f_1^2(j;t) = \frac{A_{(2,2)}(1,j;t)}{A_{(2,2)}(0,0;t)} \; ; \; t = 0, 1, 2, \ldots, 38; \quad j = 0, 1.$$

D. THE EQUATION FOR $Z_3(j;t)$

$$(6.4) \qquad Z_3(j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{2} f_3^3(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} \; ;$$

for $t = 3, \ldots, 40; \quad j = 0, 1, 2, 3$; where

$$f_3^3(i,j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^3(\mu,\nu;t-1) \binom{\mu}{1} \binom{\mu+\nu}{j} \cdot \frac{A_{(1,1)}(\mu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\nu,j;t)}{A_{(1,2)}(0,0;t)} \; ;$$

for $t = 2, \ldots, 39$; $i = 0, 1$; $j = 0, 1, 2$; where

$$f_2^3(i,j;t) = \sum_{\mu=0}^{1} f_1^3(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(2,2)}(\mu,i;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\mu,j;t)}{A_{(2,3)}(0,0;t)} ;$$

for $t = 1, 2, \ldots, 38$; $i = 0, 1$; $j = 0, 1$; where

$$f_1^3(j;t) = \frac{A_{(3,3)}(i,j;t)}{A_{(3,3)}(0,0;t)} ; \quad t = 0, 1, 2, \ldots, 37; \quad j = 0, 1.$$

E. THE EQUATION FOR $Z_4(j;t)$

(6.5)
$$Z_4(j;t) = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} f_4^4(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} ;$$

for $t = 4, 5, \ldots, 40$; $j = 0, 1, 2, \ldots, 16$; where

$$f_4^4(i,j;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{2} \sum_{\rho=0}^{1} f_3^4(\nu,\mu,\rho;t-1) \binom{\nu+\mu}{i} \binom{\mu+\rho}{j} \cdot \frac{A_{(1,1)}(\mu+\nu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\rho,j;t)}{A_{(1,2)}(0,0;t)} ;$$

for $t = 3, 4, \ldots, 39$; $i = 0, 1, 2, 3$; $j = 0, 1, 2, 3$; where

$$f_3^4(i,j,k;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^4(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \binom{\nu}{k} \cdot \frac{A_{(2,1)}(\mu,i;t)}{A_{(2,1)}(0,0;t)} \cdot \frac{A_{(2,2)}(\mu+\nu,j;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\nu,k;t)}{A_{(2,3)}(0,0;t)} ;$$

for $t = 2, 3, \ldots, 38$; $i = 0, 1$; $j = 0, 1, 2$; $k = 0, 1$; where

$$f_2^4(i,j;t) = \sum_{\mu=0}^{1} f_1^4(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(3,2)}(\mu,i;t)}{A_{(3,2)}(0,0;t)} \cdot \frac{A_{(3,3)}(\mu,j;t)}{A_{(3,3)}(0,0;t)} ;$$

for $t = 1, 2, \ldots, 37$; $i = 0, 1$; $j = 0, 1$; where

$$f_1^4(j;t) = \frac{A_{(4,3)}(1,j;t)}{A_{(4,3)}(0,0;t)} ; \quad t = 0, 1, \ldots, 36; \quad j = 0, 1$$

F. THE EQUATION FOR $Z_5(j;t)$

$$(6.6)\qquad Z_5(j;t) = \sum_{\mu=0}^{4}\sum_{\nu=0}^{6} f_5^5(\mu,\nu;t-1)\binom{\mu+\nu}{j}\cdot\frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)}\ ;$$

for $t = 5, 6, \ldots, 40$; $j = 0, 1, 2, \ldots, 10$; where

$$f_5^5(i,j;t) = \sum_{\nu=0}^{1}\sum_{\mu=0}^{3}\sum_{\rho=0}^{3} f_4^5(\nu,\mu,\rho;t-1)\binom{\mu+\nu}{i}\binom{\mu+\rho}{j}\cdot\frac{A_{(1,1)}(\mu+\nu,i;t)}{A_{(1,1)}(0,0;t)}\cdot\frac{A_{(1,2)}(\mu+\rho,j;t)}{A_{(1,2)}(0,0;t)}\ ;$$

for $t = 4, 5, 6, \ldots, 39$; $i = 0, 1, 2, 3, 4$; $j = 0, 1, 2, \ldots, 6$; where

$$f_4^5(i,j,k;t) = \sum_{\nu=0}^{1}\sum_{\mu=0}^{2}\sum_{\rho=0}^{1} f_3^5(\nu,\mu,\rho;t-1)\binom{\nu}{i}\binom{\nu+\mu}{j}\binom{\mu+\rho}{k}\cdot\frac{A_{(2,1)}(\nu,i;t)}{A_{(2,1)}(0,0;t)}\cdot\frac{A_{(2,2)}(\nu+\mu,j;t)}{A_{(2,2)}(0,0;t)}\cdot\frac{A_{(2,3)}(\mu+\rho,k;t)}{A_{(2,3)}(0,0;t)}\ ;$$

for $t = 3, \ldots, 38$; $i = 0, 1$; $j = 0, 1, 2, 3$; $k = 0, 1, 2, 3$; where

$$f_3^5(i,j,k;t) = \sum_{\nu=0}^{1}\sum_{\mu=0}^{1} f_2^5(\mu,\nu;t-1)\binom{\mu}{i}\binom{\mu+\nu}{j}\binom{\nu}{k}\cdot\frac{A_{(3,2)}(\mu,i;t)}{A_{(3,2)}(0,0;t)}\cdot\frac{A_{(3,3)}(\mu+\nu,j;t)}{A_{(3,3)}(0,0;t)}\cdot\frac{A_{(3,4)}(\nu,k;t)}{A_{(3,4)}(0,0;t)}\ ;$$

for $t = 2, 3, \ldots, 37$; $i = 0, 1$; $j = 0, 1, 2$; $k = 0, 1$; where

$$f_2^5(i,j;t) = \sum_{\mu=0}^{1} f_1^5(\mu,t-1)\binom{\mu}{i}\binom{\mu}{j}\cdot\frac{A_{(4,3)}(\mu,i;t)}{A_{(4,3)}(0,0;t)}\cdot\frac{A_{(4,4)}(\mu,j;t)}{A_{(4,4)}(0,0;t)}\ ;$$

for $t = 1, 2, 3, \ldots, 36$; $i = 0, 1$; $j = 0, 1$; where

$$f_1^5(j;t) = \frac{A_{(5,4)}(1,j;t)}{A_{(5,4)}(0,0;t)}; \quad t = 0, 1, 2, \ldots, 35; \quad j = 0, 1.$$

G. NUMERICAL RESULTS

When the calculations as described above were carried out for $\lambda = 1,3,5$, and each of the two capture modes, the numerical data is summarized in Tables 2, 3, 4, 5, 6, and 7, which describe the probabilities of j-copies arriving at the origin as a function of distance of origination.

distance \ number	0	1	2	3	4	5	6	7	8	9	10
0	.602	.398									
1	.757	.243									
2	.645	.302	.053								
3	.572	.335	.093	.013							
4	.387	.360	.184	.050	.008						
5	.240	.318	.250	.122	.039	.008	.001				

Table 2 Probabilities for $\lambda = 1$, Mode 1

distance \ number	0	1	2	3	4	5	6	7	8	9	10
0	.411	.589									
1	.566	.434									
2	.386	.433	.181								
3	.305	.399	.240	.056							
4	.126	.244	.308	.215	.087	.019	.002				
5	.003	.110	.200	.248	.211	.126	.054	.014	.001		

Table 3 Probabilities for $\lambda = 1$, Mode 2

number / distance	0	1	2	3	4	5	6	7	8	9	10
0	.411	.589									
1	.566	.434									
2	.386	.433	.181								
3	.305	.399	.240	.056							
4	.126	.244	.308	.215	.087	.019	.002				
5	.033	.110	.200	.248	.211	.126	.054	.014	.001		

Table 3 Probabilities for $\lambda = 1$, Mode 2

number / distance	0	1	2	3	4	5	6	7	8	9	10
0	.715	.285									
1	.808	.192									
2	.834	.143	.023								
3	.836	.147	.016								
4	.750	.211	.035	.004							
5	.659	.267	.063	.010	.001						

Table 4 Probabilities for $\lambda = 3$, Mode 1

number / distance	0	1	2	3	4	5	6	7	8	9	10
0	.579	.421									
1	.727	.273									
2	.659	.279	.062								
3	.642	.276	.073	.008							
4	.466	.338	.149	.040	.007	.001					
5	.306	.332	.223	.098	.030	.007	.001				

Table 5 Probabilities for $\lambda = 3$ Mode 2

distance \ number	0	1	2	3	4	5	6	7	8	9	10
0	.736	.264									
1	.883	.117									
2	.871	.119	.010								
3	.881	.110	.009								
4	.841	.140	.018	.001							
5	.843	.133	.022	.002							

Table 6 Probabilities for $\lambda = 5$, Mode 1

distance \ number	0	1	2	3	4	5	6	7	8	9	10
0	.569	.431									
1	.679	.321									
2	.674	.283	.043								
3	.715	.214	.064	.007							
4	.729	.169	.076	.021	.004						
5	.802	.115	.056	.020	.005	.001					

Table 7 Probabilities for $\lambda = 5$, Mode 2

We summarize the probability that at least one message gets through in Table 8.

	λ=1		λ=3		λ=5	
distance	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	.398	.589	.295	.421	.264	.431
1	.243	.434	.192	.273	.117	.321
2	.355	.514	.166	.341	.139	.326
3	.428	.695	.164	.358	.119	.285
4	.613	.874	.250	.534	.159	.271
5	.740	.967	.341	.692	.157	.198

Table 8 Probability That at Least One Message Gets Through

In Table 9 and the associated graph of Figure 2, we compare the expected number of copies of a single message which are accepted at the origin as a function of distance and λ.

	λ=1		λ=3		λ=5	
distance	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	.398	.589	.285	.421	.264	.431
1	.243	.434	.192	.273	.117	.321
2	.408	.795	.189	.403	.139	.369
3	.550	1.047	.179	.446	.128	.363
4	.784	1.960	.293	.789	.179	.400
5	1.386	3.178	.427	1.234	.183	.312

Table 9 The Expected Number of Copies

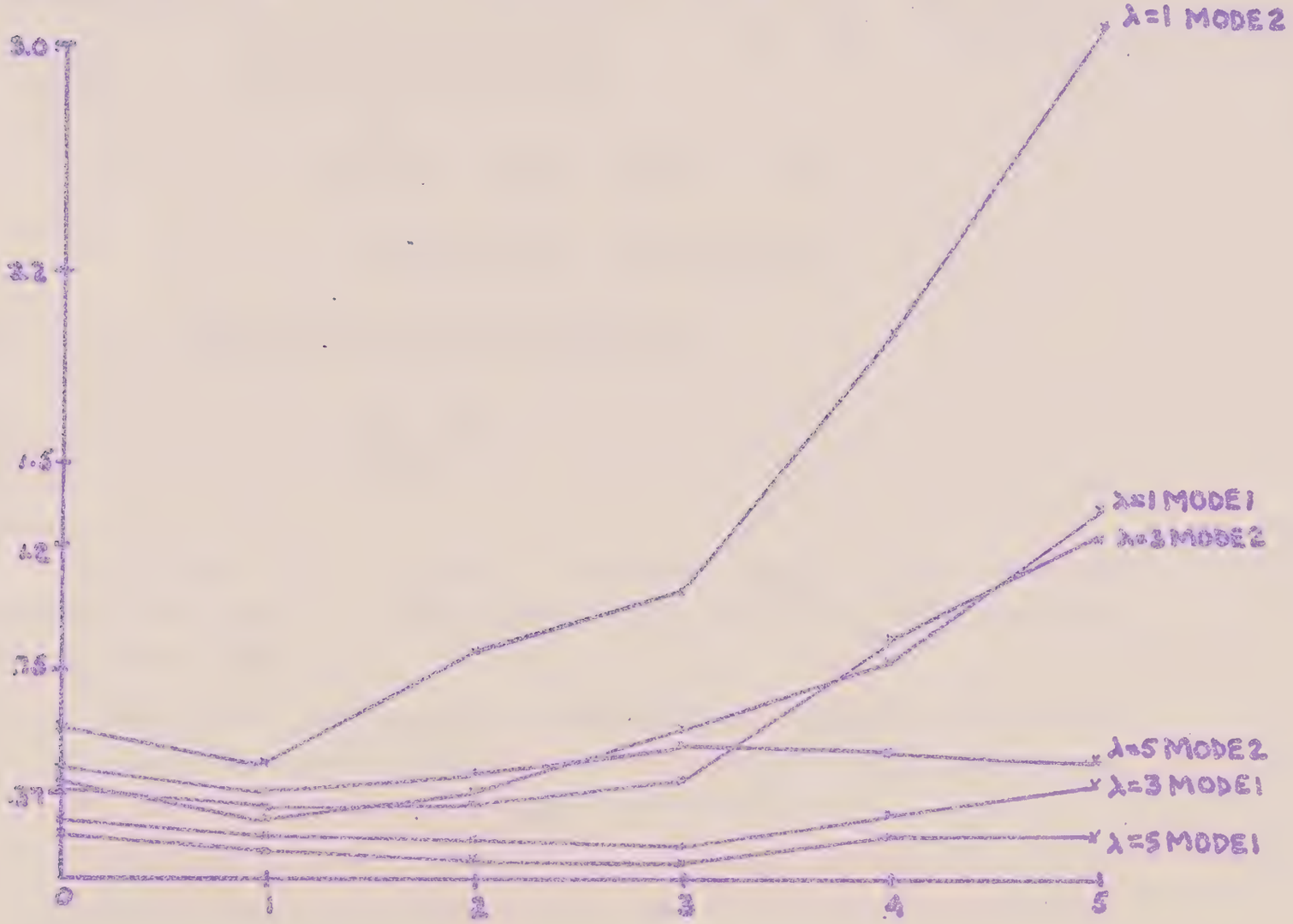

Fig. 2 Expected Number of Copies

Finally, we graph the probability of getting through as a function of distance in Figure 3.

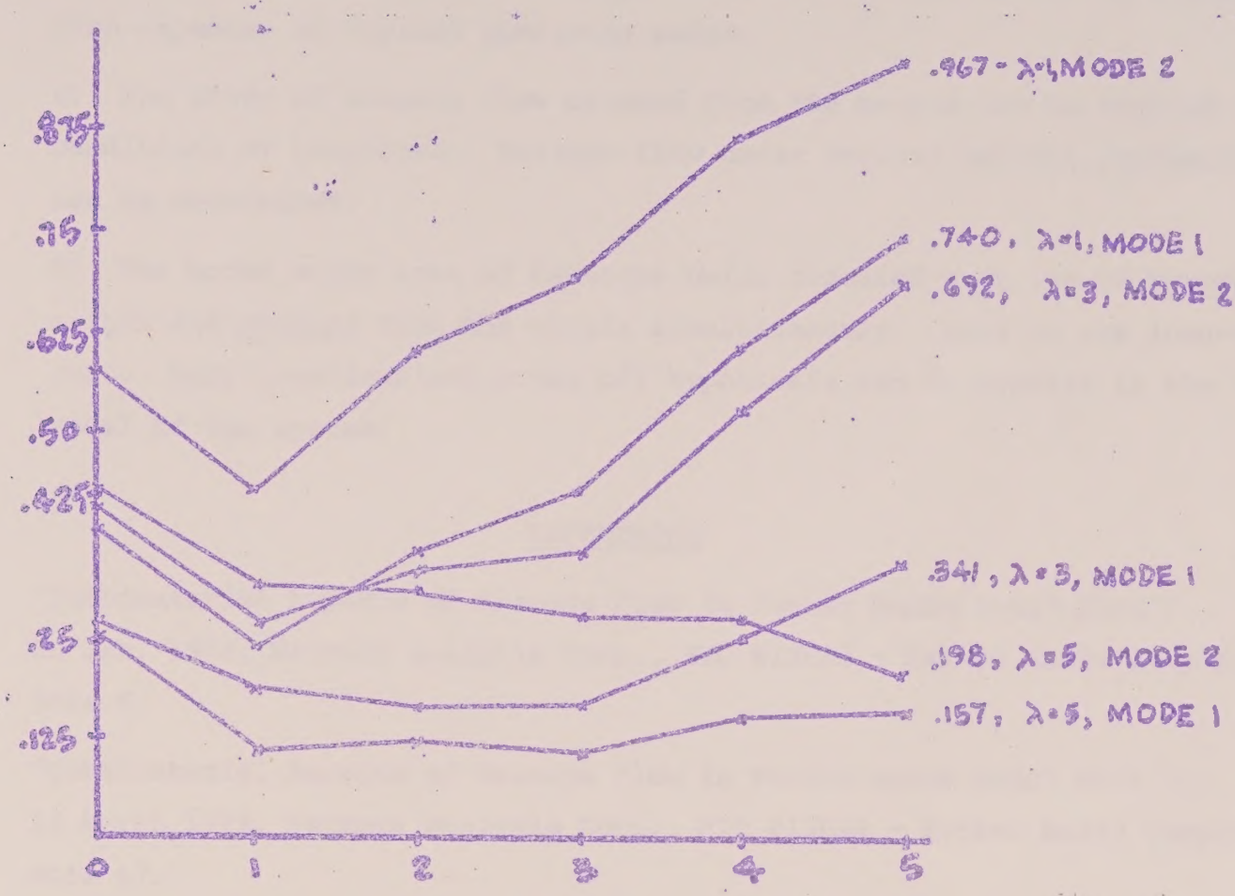

Fig. 3 Probability of Getting Through

VII SUMMARY, CONCLUSIONS, AND FUTURE PROBLEM AREAS

A cursory analysis of the data obtained indicates that when $\lambda = 5$, the net becomes essentially saturated with traffic. At saturation the probability of a message getting through to origin seems virtually constant and independent of distance.

Some further areas of extension of theoretical results combined with computer exploration and analysis will be discussed in future reports. Among the areas we will consider are:

1) The analysis of the effect of link breakdown in the net. The methods already developed can be used to determine probabilities of message survival under various partial net operation conditions.

2) The computer analysis will be used to determine the statistics of message flow outward from the origin in a fixed closed boundary configuration.

3) The results of a closed boundary analysis of message flow outward from the origin can be combined with the analysis of section four of this report to study the probability of message survival from the origin to a destination repeater at various operating modes.

4) The study of message flow outward from the origin can be studied under conditions of breakdown. Message flow under various network configurations can be determined.

5) The broad study area of messages being repeated both inward toward the origin and outward from the origin simultaneously. This is our long-range goal. Many questions and trade-off hypothesis can be studied in the full model of the system.

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